Topic 2 Factoring Trinomials of the form $ax^2 + bx + c$

Worked Example: ***Factoring Trinomials***

Factor $2x^2 + 9x + 4$

Solution:

Step One: Look for any common factors and factor first, if there are any. (there are none here!)

Step Two: Find two factors that add to b and multiply to $a \times c$

$(+2)\,(+4) = +8$

Factors that add to $+9$ and multiply to $+8$ are: $+8$ and $+1$

Step Three: Re-write the middle term using the factors found in step 2.

$$2x^2 + 8x + 1x + 4$$

Step Four: Group the first two terms and the last two terms using brackets.

$$(2x^2 + 8x) + (1x + 4)$$

Step Five: Factor the greatest common factor from the first two terms so that x^2 will be positive and the greatest common factor from the last two terms so that x will be positive.

$$2x(x + 4) + 1(x + 4)$$

Step Six: Factor the greatest common factor from the entire polynomial.

$$\boldsymbol{(x + 4)(2x + 1)}$$

Try Yourself *2.2.1: Factoring Trinomials of the form $ax^2 + bx + c$*

Fully factor the following trinomials:

b. $2x^2 - 9x - 35$

b. $6x^2 - x - 2$

Ans: a) $(2x+5)(x-7)$ b) $(3x-2)(2x+1)$

Topic 3 Factoring Trinomials of the form $af(x)^2 + bf(x) + c$

***Worked Example:* Factoring Trinomials**

Factor $(x-3)^2 + 2(x-3) - 8$

Solution:

Step One: Substitute a monomial for the common binomial expression.

$(x-3)^2 + 2(x-3) - 8$ Let $d = x - 3$

Step Two: Re-write the "new" trinomial using the monomial from step one.

$d^2 + 2d - 8$

Step Three: Now that the trinomial is written in the form $y = ax^2 + bx + c$, find two factors that add to b and multiply to $a \times c$ $(+1)(-8) = -8$

Factors that add to $+2$ and multiply to -8 are: $+4$ and -2

Step Four: Re-write the middle term using the factors found in step 2.

$d^2 + 4d - 2d - 8$

Step Five: Group the first two terms and the last two terms using brackets.

$(d^2 + 4d) + (-2d - 8)$

Step Six: Factor the greatest common factor from the first two terms so that x^2 will be positive and the greatest common factor from the last two terms so that x will be positive.

$d(d+4) - 2(d+4)$

Step Seven: Factor the greatest common factor from the entire polynomial.

$(d-2)(d+4)$

Step Eight: Replace the monomial with its value in step one and simplify each factor.

$([x-3] - 2)([x-3] + 4)$

$(x-5)(x+1)$

Try Yourself *2.3.1:* *Factoring Trinomials of the form $af(x)^2 + bf(x) + c$*

Fully factor the following trinomials:

a. $(x+1)^2 - (x+1) - 12$

b. $(2x-3)^2 - 3(2x-3) - 40$

Ans: a) $(x-3)(x+4)$ b) $(2x-11)(2x+2)$

Topic 4 Factoring Difference of squares of the form $ax^2 - by^2$

Worked Example: ***Factoring Difference of Squares***

Factor $144x^2 - 25y^2$

Solution:

Step One: *Look for any common factors and factor them first.*

Step Two: *Re-write each term of the binomial as a factor squared.*

$\mathbf{12^2x^2 - 5^2y^2}$

Step Three: Place the first factor as the first term in each bracket and the second factor as the second term in each bracket. In one set of the brackets place an addition symbol in between the terms and in the other set of brackets place a subtraction symbol in-between the terms.

$= \mathbf{(12x - 5y)(12x + 5y)}$

Try Yourself 2.4.1: Factoring binomials of the form $ax^2 - by^2$

Fully factor the following binomials:

a) $144a^2 - 25b^2$

b) $49x^2y^4 - 16$

Ans: a) $(12a - 5b)(12a + 5b)$
b) $(7xy^2 - 4)(7xy^2 + 4)$

Try Yourself 2.4.2: Factoring Trinomials of the form $af(x)^2 - bf(x)^2$

Fully factor $(2x + 1)^2 - 64$

Ans: $(2x - 7)(2x + 9)$

Try Yourself 2.4.3: Factoring binomial of the form $af(x)^2 - bf(y)^2$

Fully factor the following binomials

a) $x^2 - (v + z)^2$

b) $3(x + 5)^2 - 25(y - 3)^2$

Ans: $(x - v - z)(x + v + z)$

b) $4(x + 5)^2 - 25(y - 3)^2$

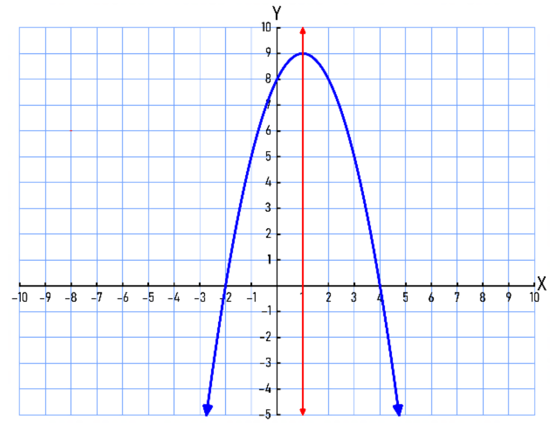
Ans: $(2x - 5y + 25)(2x + 5y - 5)$

Unit 3 – *RTD Math 20-1*

Quadratic Functions and Equations 3

Topic 1 Quadratic Functions

- polynomial of degree two (the largest exponent of the variable is 2)
- the **general form** is $ax^2 + bx + c$
- the **standard form** is $a(x - p)^2 + q$



Vertex: the point (1, 9 in our example)
- midpoint of the graph; the highest or lowest point of the graph

Maximum/Minimum: the highest/lowest point a graph reaches
- the y-coordinate of the vertex; ($y = 9$ in our example)

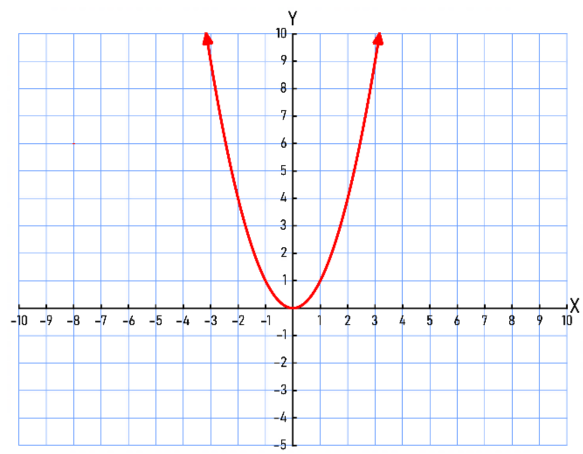
Domain: the set of all values of the independent variable (x)
- the arms of a parabola extend infinitely in both directions covering
  all possible x-values so D: $\{x \epsilon \mathbb{R}\}$

Range: the set of all values of the dependent variable (y)
- graphs opening up have a minimum value so R: {y ≥ minimum}
- graphs opening down have a maximum value so
  R: {y ≤ maximum} R: $\{y \leq 9\}$ in our example

Axis of symmetry: a vertical line that bisects (cuts in half) the parabola
- passes through the x-coordinate of the vertex; (red line, $x = 1$ in our example)

y-intercept: the point the graph crosses the y-axis
- (0, 8) in our example

x- intercepts: the points where the graph crosses the x-axis
- $(x, 0)$ $(-2, 0)$ and $(4, 0)$ in our example



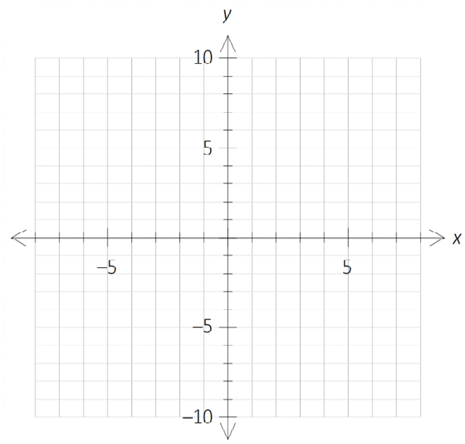
Base Function $y = x^2$

- All quadratic functions can be compared to the base function. $y = x^2 = 1x^2 + 0x + 0$
- By changing the values of a, b and c in $ax^2 + bx + c$ we we can stretch and slide the graph around the coordinate system.

Topic 2 Effects of coefficients on the quadratic function

Try Yourself *3.2.1: Effect of the sign a on $y = ax^2$*

Graph both functions on the same grid and summarize your findings.



$y = 2x^2$

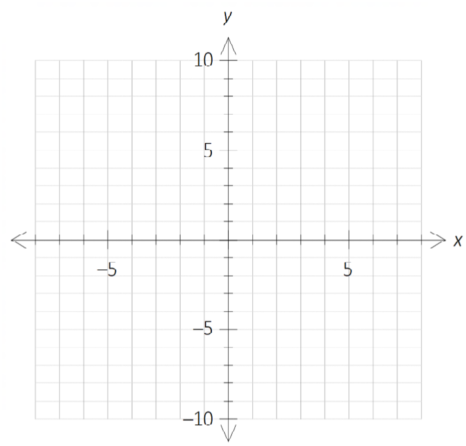
$y = -2x^2$

Conclusion:

Ans: The negative sign reverses the direction of opening

Try Yourself *3.2.2: Effect of the size a on $y = ax^2$*

Graph both functions on the same grid and summarize your findings.



$y = \frac{1}{8}x^2$

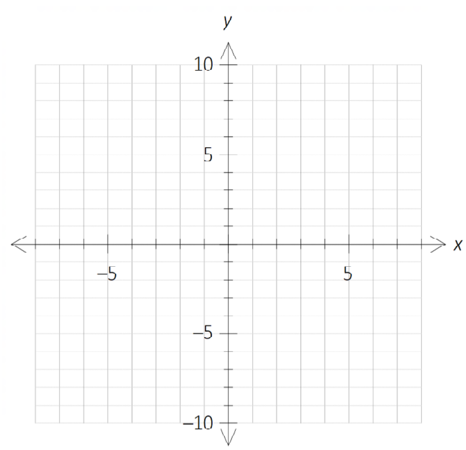
$y = 8x^2$

Conclusion:

Ans: The constant controls the wide of the graph (alternatively, how fast the y values increase.

Try Yourself *3.2.3: Effect of q on $y = x^2 + q$*

Graph both functions on the same grid and summarize your findings.



$y = x^2 + 3$

$y = x^2 - 3$

Conclusion:

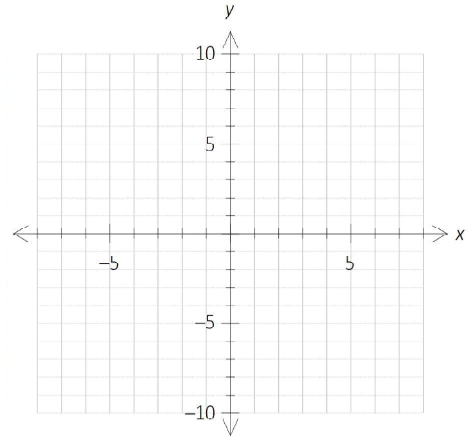
Ans: The constant value 3 shifts (“translates”) the graph up or down depending on the sign.

Try Yourself 3.2.4: *Effect of q on $y = (x-p)^2$*

Graph both functions on the same grid and summarize your findings.

$y = (x-4)^2$

$y = (x+4)^2$



Conclusion:

Ans: The constant value 4 shifts ("translates") the graph left or right depending on the sign.

Summary of the effects of "a", "p", and "q":

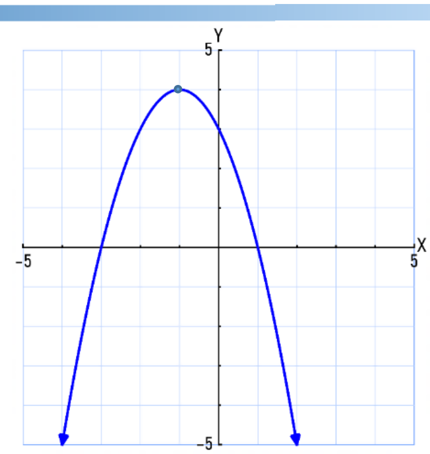
- the sign of a determines whether the parabola opens up ($a > 0$) or down ($a < 0$) and the magnitude (size) of a itself tells us how wide or narrow the parabola is
  - if the magnitude of a is less than 1 ($0 < |a| < 1$) the parabola is wider than the base function, $y = x^2$
  - if the magnitude of a is greater than 1 ($|a| > 1$) the parabola is narrower than the base function, $y = x^2$
- q determines whether a parabola moves up ($q > 0$) or down ($q < 0$)
- p determines whether a parabola moves right ($x - p$) or left ($x + p$)

Topic 3 Vertex Form

$y = a(x-p)^2 + q$ where:

- (p, q) is the vertex
- $x = p$ is the axis of symmetry
- $y = q$ is the maximum/minimum value

Try Yourself 3.3.1: *State the equation given that $|a| = 1$*

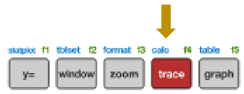


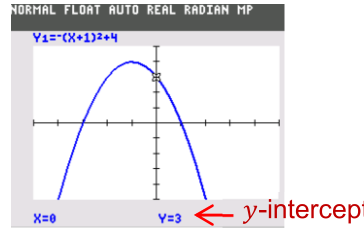
Ans: $y = -(x+1)^2 + 4$

Tips to finding x and y intercepts on the graphing calculator:

y-intercept

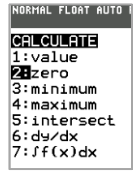
- enter equation into **Y=** — Go to **graph…trace**
- enter 0 and the y-intercept will show.

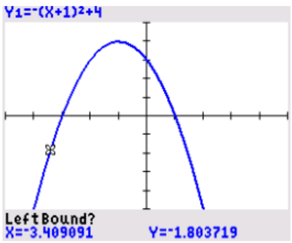


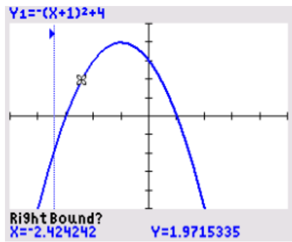


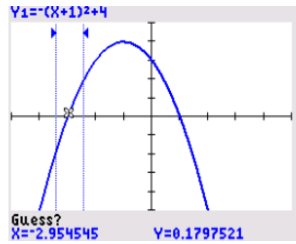
x-intercept(s)

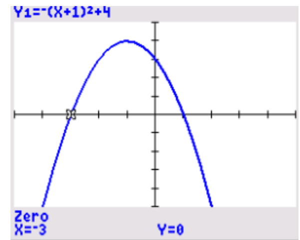
- enter the equation into **Y=**
- go to the **calc** menu (2nd trace), and choose **option 2 (zero)**











- move cursor to the left of the *x*-intercept (left bound) and press enter
- move cursor to the right of the *x*-intercept (right bound) and press enter twice (including once for "guess")
- zero shown on the bottom of the screen is the *x*-intercept of the function

Try Yourself 3.3.2: Identify important parts of a quadratic in vertex form without graphing

Fill in the table below

	$y = ¼(x-1)^2 - 2$	$y = 2(x+4)^2 + 1$
Vertex:	______________	______________
Max/Min:	______________	______________
Axis:	______________	______________
Domain:	______________	______________
Range:	______________	______________
***x*-intercepts:**	______________	______________
***y*-intercepts:**	______________	______________

$Ans: (1,-2), -2\ min, x = 1, x \in \mathbb{R}, y \geq -2, x = 1 \pm 2\sqrt{2}, y = -1.75$

$Ans: (-4,1), 1\ min, x = -4, x \in \mathbb{R}, y \geq 4, x = DNE, y = 33$